Identity laws	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	$A \cap \mathcal{U} = A$ $A \cup \varnothing = A$
Domination laws	$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	$A \cup \mathcal{U} = \mathcal{U}$ $A \cap \varnothing = \varnothing$
Idempotent laws	$p \wedge p \equiv p$ $p \vee p \equiv p$	$A \cap A = A$ $A \cup A = A$
Double negation / complement law	$\neg(\neg p) \equiv p$	$\overline{(\bar{A})} = A$
Commutative laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	$A \cap B = B \cap A$ $A \cup B = B \cup A$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
De Morgan's laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\overline{A \cap C} = \bar{A} \cup \bar{C}$ $\overline{A \cup C} = \bar{A} \cap \bar{C}$
Absorption laws	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	$A \cap (A \cup B) = A$ $A \cup (A \cap B) = A$
Negation / complement laws	$p \wedge \neg p \equiv \mathbf{F}$ $p \vee \neg p \equiv \mathbf{T}$	$A \cap \bar{A} = \varnothing$ $A \cup \bar{A} = \mathcal{U}$

Table 1: Equivalence Laws

$p \to q \equiv \neg p \vee q \equiv\neq q \to \neg p$	$p \leftrightarrow (p \to q) \wedge (q \to p) \equiv \neg p \leftrightarrow \neg q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$p \vee q \equiv \neg p \to q$	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$
$p \wedge q \equiv \neg(p \to \neg q)$	
$\neg(p \to q) \equiv p \wedge \neg q$	
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$	
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$	
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$	
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$	

Table 2: Equivalences to Conditionals and Biconditionals

$\neg\exists x P(x)$	$\forall x \neg P(x)$
$\neg\forall x P(x)$	$\exists x \neg P(x)$

Table 3: De Morgan's Laws for Quantifiers

$\forall x \forall y P(x, y) \equiv \forall y \forall x P(x, y)$
$\forall x \exists y P(x, y) \equiv \neg[\exists x \forall y \neg P(x, y)]$
$\exists x \forall y P(x, y) \equiv \neg[\forall x \exists y \neg P(x, y)]$
$\exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$

Table 4: Quantifiers of Two Varaibles

Modus ponens	p $\underline{p \to q}$ $\therefore q$
Modus tollens	$\neg q$ $\underline{p \to q}$ $\therefore \neg p$
Hypothetical syllogism	$p \to q$ $\underline{q \to r}$ $\therefore p \to r$
Disjunctive syllogism	$p \vee q$ $\underline{\neg p}$ $\therefore q$
Addition	$\underline{p}$ $\therefore p \vee q$
Simplification	$\underline{p \wedge q}$ $\therefore p$
Conjunction	p $\underline{\neg q}$ $\therefore p \wedge q$
Resolution	$p \vee q$ $\underline{\neg p \vee r}$ $\therefore q \vee r$
Universal instantiation	$\underline{\forall x P(x)}$ $\therefore P(c)$
Universal generalization	$\underline{P(c) \text{ for an arbitrary } c}$ $\therefore \forall x P(x)$
Existential instantiation	$\underline{\exists x P(x)}$ $\therefore P(c)$ for some element c
Existential generalization	$\underline{P(c) \text{ for some element } c}$ $\therefore \exists x P(x)$
Universal modus ponens	$\forall x \in D, (P(x) \to Q(x))$ $\underline{P(a) \text{ where } a \in D}$ $\therefore Q(a)$
Universal modus tollens	$\forall x \in D, (P(x) \to Q(x))$ $\underline{\neg Q(a) \text{ where } a \in D}$ $\therefore \neg P(a)$

Table 5: Rules of Inference

Types of Proofs

Proof of Validity

- Direct Proof (Show $p \to q$)
  - hypothetical syllogism
  - vacuous proof (p is false)
  - trivial proof (q is true)
- Indirect Proof

  - proof by contrapositive (Show $\neg q \to \neg p$ to show $p \to q$)
  - proof by contradiction (Show $\neg p \to \mathbf{F}$ to show p)
  - disproof by counterexample

Proof of Equivalence (Show $p \leftrightarrow q$)

Generalization Proof (Show $\forall x \in D, P(x)$)

- exhaustive proof (Show $P(x_i)$ for all $x_i \in D$)
- proof by cases (Show $\forall x \in D_i, P(x)$, for a partition of D, $\{D_i\}$)
- without loss of generality (Showing for multiple cases by common properties)

Existence Proof (Show $\exists x \in D, P(x)$)

- constructive proof (Determine $x \in D$ such that $P(x)$)
- nonconstructive proof (Imply $x \in D$ exists such that $P(x)$ by context)

Uniqueness Proof (Show $\exists! x \in D, P(x)$)

Inductive Proof (Show $\forall n \in \mathbb{N}, P(n)$)

- weak induction (Show $P(1)$ and $P(n) \to P(n+1)$)
- strong induction (Show $P(1), \ldots, P(k)$ and $P(1) \wedge P(2) \wedge \cdots \wedge P(n) \to P(n+1)$)
- structural induction (Induction on a recursive function / structure)

Some Fallacies (errors in logic / proof)

- circular argument
- affirming the conclusion (see modus ponens)
- denying the hypothesis (see modus tollens)
- begging the question