

The sheets attached to the questionnaire can be used for scratch-work—if so, write name and attach to submission

Some Formulas

$$\left| \int_a^b f(x) dx - \frac{b-a}{n} \left[\sum_{k=1}^{n-1} f\left(a + k \frac{b-a}{n}\right) + \frac{f(a) + f(b)}{2} \right] \right| < \frac{M(b-a)^3}{12n^2} \quad \text{where } M = \max_{a \leq \xi \leq b} |f''(\xi)|;$$

if $0 \leq b_{n+1} < b_n$ for all n and $\lim_{n \rightarrow \infty} b_n = 0$, then $\left| \sum_{n=N+1}^{\infty} (-1)^n b_n \right| = \left| \sum_{n=0}^{\infty} (-1)^n b_n - \sum_{n=0}^N (-1)^n b_n \right| < b_{N+1}$;

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1; \quad \tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)}, |x| \leq 1; \quad \ln \frac{1}{1-x} = \sum_{n=1}^{\infty} \frac{x^n}{n}, -1 \leq x < 1;$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n; \quad \binom{k}{n} = \frac{k}{n} \cdot \frac{k-1}{n-1} \cdots \frac{k-n+2}{2} \cdot \frac{k-n+1}{1}, |x| < 1; \quad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!};$$

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}; \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n; \quad T_N(x) = \sum_{n=0}^N \frac{f^{(n)}(a)}{n!} (x-a)^n; \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!};$$

$$|R_N(x)| = |f(x) - T_N(x)| \leq \frac{M}{(N+1)!} |x-a|^{N+1}, \quad \text{where } M = \max |f^{(N+1)}(\xi)|, \xi \text{ between } x \text{ and } a;$$

if $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n, |x-a| < R$, then $\int f(x) dx = \sum_{n=0}^{\infty} \bar{c}_n (x-a)^n, |x-a| < R, \bar{c}_n = \frac{c_{n-1}}{n}, n > 1$;

$$\left| \int_a^b f(x) dx - \frac{b-a}{3n} \left[f(a) + 2 \sum_{k=1}^{n-1} f\left(a + \frac{b-a}{n} k\right) + 2 \sum_{k=1}^{(n/2)-1} f\left(a + \frac{b-a}{n} (2k-1)\right) + f(b) \right] \right| < \frac{M(b-a)^5}{180n^4}$$

$$\text{where } M = \max_{a \leq \xi \leq b} |f^{(4)}(\xi)|$$

VIOLATIONS WILL RESULT IN ZERO MARKS FOR THIS TEST
DO NOT TURN OVER THESE SHEETS OR VIEW TEST QUESTIONS BEFORE 09:05

Reminders

On your workdesk:

- Writing implements: pens, correction devices
- A water bottle (optional)
- Tissues (optional)

Phones, tablets or other internet enabled devices should be **turned off** (not silent mode), and **inside your bag**

No communication with other test-takers allowed: raise hand for attention and blank sheets

Bags should be placed on either side of the room

No toilet breaks during test-taking

Write your name on pages 1 and 3 of the questionnaire

Submit sheets to front
BEFORE clock is at 10:45:
Late sheets will NOT be accepted